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A new Rectangle Subsections Method (RSM) for the design of a reinforced concrete T-shaped cross-section bridge girders

Nowa Metoda Podprzekrojów Prostokątnych (RSM) projektowania żelbetowych przekrojów teowych dźwigarów mostowych

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Abstract. The aim of this article is to present the new approach to a design of t-shaped beam elements, called a Rectangle Subsections Method (RSM). Its biggest advantage is that the internal forces from the components do not need to be added to obtain the total forces. In case of sagging, the results for RSM and Traditional Design Method (TM) appeared similar. At hogging, the RSM was more conservative – the ratio of the designed to the tested reinforcement area was 3.3 in RSM vs 2.6 in TM.

Keywords: concrete; structures; finite elements; bridges; design.

Streszczenie. Celem pracy jest prezentacja nowego podejścia do projektowania przekrojów teowych w elementach belkowych, zwanego Metodą Podprzekrojów Prostokątnych (RSM). Jego główną zaletą jest brak konieczności dodawania sił wewnętrznych z komponentów w celu otrzymania sił wypadkowych. W przypadku momentów dodatnich wyniki RSM i Metody Tradycyjnej (TM) okazały się zbieżne. Przy zginaniu ujemnym RSM była bardziej konserwatywna – stosunek powierzchni zbrojenia zaprojektowanego do badanego wynosił 3,3 w RSM vs 2,6 w TM.

Słowa kluczowe: beton; konstrukcje; elementy skończone; mosty; projektowanie.

The following paper presents a new approach to a design of reinforced concrete tee sections. Such cross-sections are widely used in bridge structures, where a deck slab is joined with a grillage of longitudinal and transversal girders. The t-shaped cross-sections are well tested by many researchers [1 – 5]. Special cases were also investigated. For instance, beams without stirrups [6], with holes along its length [7] or with non-prismatic flanges [8]. Also several methods were developed to effectively calculate elements with such sections [9 – 12].

Current design practice cannot omit computational methods, especially finite element method, widely used by engineers. Traditional designing methods need to be adjusted, so that they can be used with these numerical algorithms. For instance, slab integrated with beams supporting it, can be modelled in 2 ways: either using 2 – dimensional plane elements for slab and 1 – dimensional elements with rectangular cross section for beams (BPM – Beam/Plate Model, Figure 1 and Figure 2) or, using only 1-dimensional elements with tee cross sections (GM – Grillage Model, Figure 3 and Figure 4). Each of these approaches has its advantages and flaws. BPM allows a designer using one model for the whole process and extracting all the ne-

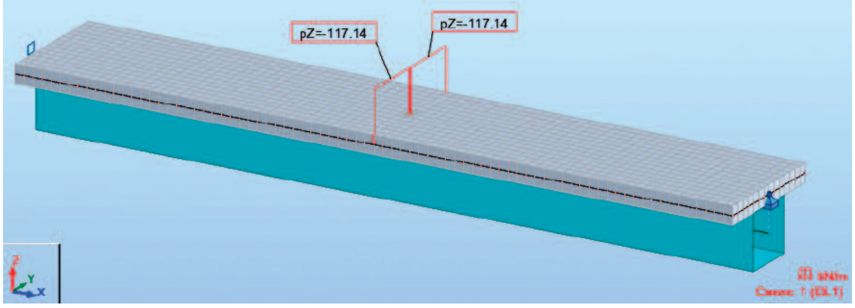

Fig. 1. Beam/Plate finite element model for sagging test beam
Rys. 1. Model MES belkowo-płytowy w przypadku testowanej belki zginanej momentem dodatnim

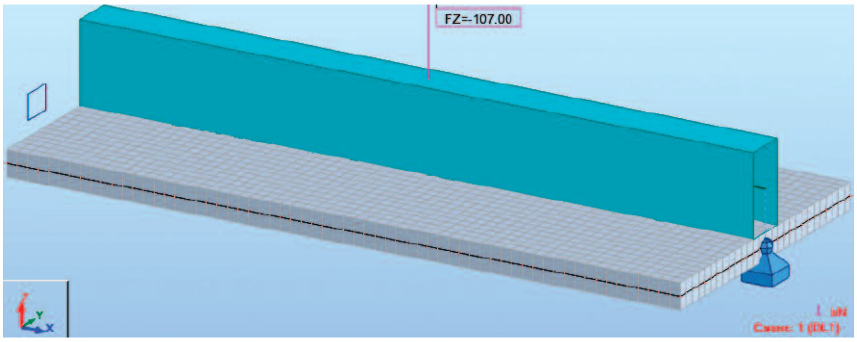

Fig. 2. Beam/Plate finite element model for hogging test beam
Rys. 2. Model MES belkowo-płytowy w przypadku testowanej belki zginanej momentem ujemnym

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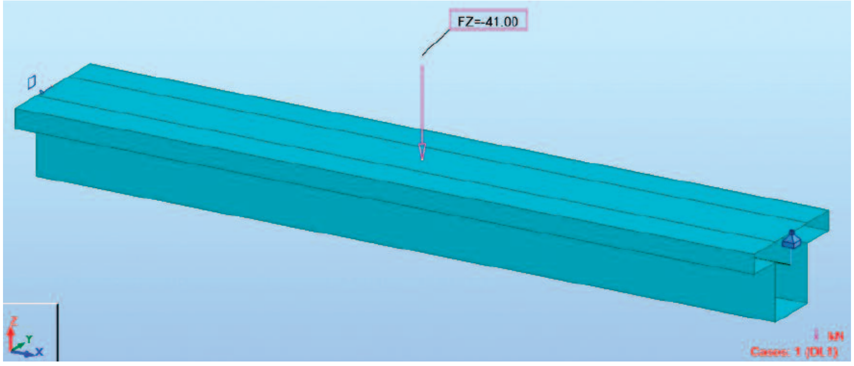

Fig. 3. Grillage finite element model for sagging test beam
Rys. 3. Model MES rusztowy w przypadku testowanej belki zginanej momentem dodatnim

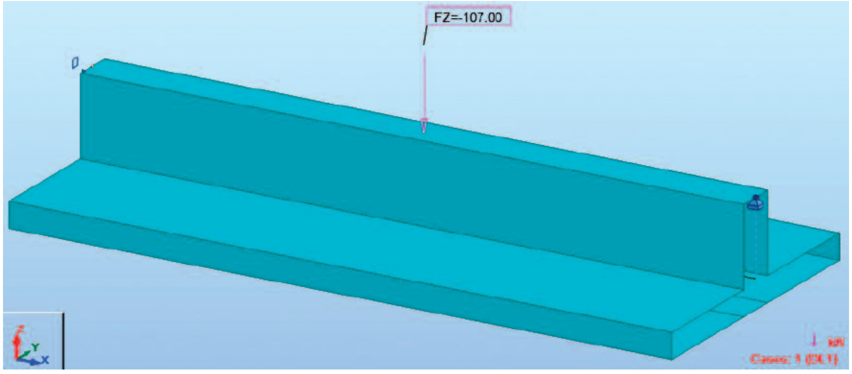

Fig. 4. Grillage finite element model for hogging test beam
Rys. 4. Model MES rusztowy w przypadku testowanej belki zginanej momentem ujemnym

cessary information from it. Meanwhile, GM forces the designer to make another model for the purpose of designing transversal reinforcement in the slab. It is implied by the fact that the former model usually uses diaphragm to distribute a load from the slab to the beam elements. In fact, to be able to design the deck as a set of tee sections, one has to create the two above mentioned models. Another issue with GM is an additional mass of girders crossing each other, which often needs to be manually subtracted from the model. Of, course, there are also some advantages of this approach. Firstly, the cross-section design procedure seems to be simpler. This is due to a lack of a longitudinal force in the cross section (omitting negligible external forces) and due to having only one common t-shaped cross section to be designed instead of two rectangle subsections, which need to be converted to tee section before design. Secondly, there is no need to integrate the internal forces across the girder, as it has to be done in case of BPM. Yet, it is not a big inconvenience, since most of the modern finite element software packages allow the designer to integrate automatically over an arbitrary cross-section (see Figure 5).

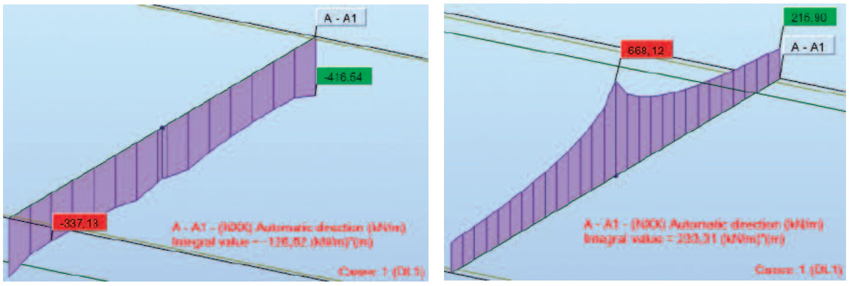

Fig. 5. The distribution of the membrane force within a slab
Rys. 5. Rozkład siły membranowej w płycie

On the other hand, having only one finite element model (BPM), one saves time on preparing the additional model for calculating transversal reinforcement in slab, but the need for extra steps during design emerges.

The traditional approach relies on adding bending moments occurring in the plate and the beam, to the moment balanced by the longitudinal forces present in both components and then design the tee section for that moment [13] (TM – *Traditional Method*). The new approach presented here suggests to design the components separately, taking into account bending moments and respective longitudinal forces calculated for these components (RSM – Rectangle Subsections Method). This study covers the design calculations of some sample tee beams, using these two approaches. It also uses an experimental data from available papers [14, 15].

Two cases need to be distinguished for the design purposes: a case where the slab is predominantly influenced by the tensile stresses (typical cross section in the vicinity of intermediate supports of a bridge deck) and a case where the slab is in compression (typically middle span cross sections).

Materials and methods

A subject of the analyses were some reinforced concrete girders, consisting of a slab, joined monolithically with stringer. Both these elements together create prismatic bar of tee cross section.

Slabs were modelled using 2 – dimensional planar quadrilateral 4-node finite elements with 6 degrees of freedom in each node. In turn, beam elements were created using 1-dimensional elements with 2 nodes per element. Average element size was 25 mm.

Static calculations were performed assuming linear elastic analysis. Steel reinforcement was not taken into account during static calculations phase. All the significant phenomena present in the concrete (e.g., creep, shrinkage, cracking, tension stiffening), were taken into account at a post static analysis stage, according to Eurocode 2 [16].

For the analysis of sagging (positive moment bending, i. e., flanges in compression), it has been assumed a simply supported beam, subjected to a 3-point bending. A span length was 2 meters and the load was applied vertically down at the middle of the span. A slab was 50 mm thick and the web of the beam was 150 mm wide and 150 mm high. Totalthus a total cross-sectional height was 200 mm. Flange width was 350 mm. Concrete compressive strength (cylindrical) was assumed 33 MPa. Yield strength of reinforcing steel was taken 550 MPa. Longitudinal reinforcement in a test beam [14] constituted 2 rebars of 12 mm diameter. A transverse reinforcement was in the form of stirrups, 8 mm in diameter, spread with 200 mm space. During the test, the beam collapsed under a concentrated load of 41 kN, which gives bending moment of 20.5 kNm. At the failure, the beam deflected by 15 mm and apparent crack widths were between 6 and 11 mm. Thegeometry and the dimensions of a cross section, as well as the test beam reinforcement distribution, are summarizedin Figure 6.

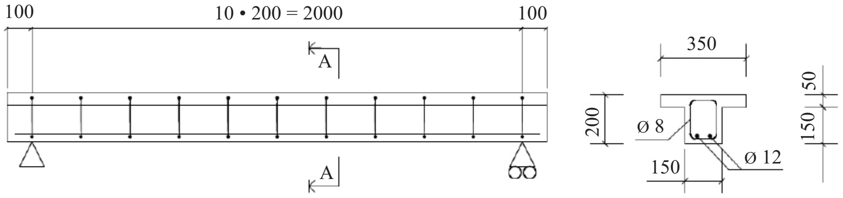

Fig. 6. The geometry and the dimensions of a test beam loaded with sagging moment [14]
Rys. 6. Geometria i wymiary testowej belki obciążonej momentem dodatnim [14]

Hogging (negative moment bending, i.e., flanges in tension) has also been analyzed on a simply supported beam, subjected to a 3-point bending. This time the span length was 1.6 meters and the load was also applied at the middle of the span. A total height of a cross-section was 250 mm, a slab was 60 mm thick and 700 mm wide. A web was 100 mm wide and 190 mm high. Concrete compressive strength (cylindrical) was assumed 25 MPa. Yield strength of reinforcing steel was taken 420 MPa. Longitudinal reinforcement of a test beam [15] constituted 6 rebars of 8 mm in diameter within the flange and 2 rebars of 6 mm in diameter on the opposite side of the beam, in the web. The transverse reinforcement was in the form of stirrups, 6 mm in diameter, spaced by 75 mm. During the test, the beam collapsed under a concentrated load of 107 kN, which gives bending moment of 42.8 kNm. At failure, the beam deflected by 11.4 mm. Crack width at failure are not available. The geometry and the dimensions of a cross section, as well as the test beam reinforcement distribution, are summarizedin Figure 7.

It needs to be noted that the tested beams do not fully conform to the code. For instance beam from the Figure 6 should also contain longitudinal bars in the upper part of the section and the beam depicted in Figure 7 should have bigger spacings between bottom longitudinal bars. But this inconsistency did not influence results considerably, therefore the issue was omitted.

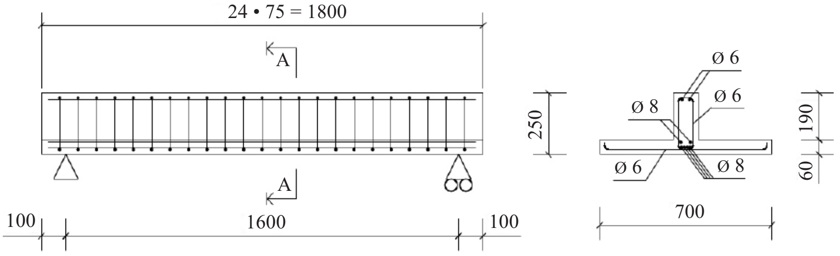

Fig. 7. The geometry and the dimensions of a test beam loaded with hogging moment [15]
Rys. 7. Geometria i wymiary testowej belki obciążonej momentem ujemnym [15]

Results

Sagging (positive bending moment, flanges compressed). A difference in obtained the required reinforcement cross-sectional area due to Ultimate Limit State (ULS), for RSM is 244 mm^2, whereas for TM – 222 mm^2. A difference was 9% of the higher value. The RSM gives more conservative value. A comparison of Serviceability Limit State (SLS) is gathered in Table 1. It contains results for amount of reinforcement required due to ULS for each approach (222 and 244 mm^2) as well as for a case of 3 longitudinal bars at the bottom of the beam (339 mm^2) and for the amount of reinforcement which should be used to fulfill all the code requirements including SLS (588 mm^2). The decisive ratios are marked bold. Comparison using simplified method of deflection check is based on rectangular stringer span/depth limit calculation alone (without slab) versus whole t-shaped section limit.

Hogging (negative bending moment, flanges in tension). A difference in obtained the required cross-sectional area due to ULS for RSM is 555 mm^2, whereas for TM – 601 mm^2. A difference was 8% of the higher value. The TM method gives more conservative value. A comparison of SLS is gathered in Table 2. It contains results for amount of reinforcement required due to ULS for each approach (555 and 601 mm^2) as well as for the amount of reinforcement which

Table 1. Sagging results
Tabela 1. Wyniki w przypadku zginania dodatniego

Amount of reinforcement [mm^2]	Concrete stresses [MPa]		Steel stresses [MPa]		Crack width [mm]		Span/depth limit due to deflection		Deflection including linear creep [mm]		Deflection excluding creep [mm]	
	RSM	TM	RSM	TM	RSM	TM	RSM	TM	RSM	TM	RSM	TM
222	4.74	7.56	521	468	0.25	0.26	9.5	11.75	9.95	15.9	2.2	3.37
244	4.88	7.01	483	434	0.22	0.23	10	12.7	9.6	14.8	2.1	3.13
339	5.37	5.49	375	340	0.15	0.16	11.8	16.2	8.6	11.55	1.9	2.45
588	6.12	3.78	244	234	0.08	0.09	16.38	23.5	7.6	7.95	1.65	1.7
Amount of reinforcement [mm^2]	**Ratio of concrete stress limit [%]**		**Ratio of steel stress limit [%]**		**Ratio of crack limit [%]**		**Ratio of span/depth limit [%]**		**Ratio of deflection limit including linear creep [%]**			
	RSM	**TM**	**RSM**	**TM**	**RSM**	**TM**	**RSM**	**TM**	**RSM**	**TM**		
222	14	23	118	106	83	87	170	98	124	199		
244	15	21	110	99	73	77	161	91	120	185		
339	16	17	85	77	50	53	137	71	108	144		
588	19	11	55	53	27	30	**98**	49	95	**99**		

Table 2. Hogging results

Tabela 2. Wyniki w przypadku zginania ujemnego

Amount of reinforcement [mm^2]	Concrete stresses [MPa]		Steel stresses [MPa]		Crack width [mm]		Maximum stirrups spacing [mm]	
	RSM	**TM**	**RSM**	**TM**	**RSM**	**TM**	**RSM**	**TM**
555	13	19	511.5	518.6	0.25	0.215	17,4	17,55
601	13.4	17.2	483	429	0.23	0.17		
784	14.3	15	397.6	333	0.18	0.13		
985	14.9	13.4	335.5	268	0.14	0.1		
Amount of reinforcement [mm^2]	**Ratio of concrete stress limit [%]**		**Ratio of steel stress limit [%]**		**Ratio of crack limit [%]**			
	RSM	**TM**	**RSM**	**TM**	**RSM**	**TM**		
555	52	76	152	154	83	72		
601	54	69	144	128	77	57		
784	57	60	118	**99**	60	43		
985	60	54	**100**	80	47	33		

should be used to fulfill all the code requirements including SLS for TM and RSM (784 and 985 mm^2, respectively). The decisive ratios are marked bold.

Discussion

Generally, the Serviceability Limit State rules the design in both cases of hogging and sagging. For positive moment bending, the deflection was a ruling condition. For negative one – steel stress was a crucial factor.

For sagging, using simplified method of assessing the deflection and reinforcement amount required by Eurocode 2, RSM approach yields 98 % ratio of span/depth limit, while TM yields 49% of the limit value. It means, using the simplified methodof deflection limit assessing, allows for much more efficient design with TM approach.

On the other hand, the exact method yields very similar results for both approaches: 99% of limit value (allowed deflection) for TM vs 95% for RSM. Here, the TM is slightly more conservative.

Worth noticing is that the exact method of deflection calculation in theTM seems to be more conservative than the simplified one. Creep effect is higher for TM, as it depends, among others, on the concrete stress level. This difference diminishes with an increase of reinforcement ratio, so as to finally disappear at the design level of reinforcement. At hogging, the difference in steel stress is about 20%. In this case the RSM is more conservative.

Conclusions

The article presents the new approach to the design of t-shaped beam elements, the approach called here a Rectangle Subsections Method (RSM). It relies on an independent design of the slab deck and the girder supporting it, despite their structural integrity. Such an approach results from the most common method of modelling of these elements inthefinite element software, as separate sets of finite elements of different dimension in space each. Beams are usually modelled using 1-dimensional elements, while slabsare modelled with 2-dimensional planar elements.

The biggest advantage of RSM is that the internal forces from the two components do not need to be added to each other to obtain the total effect. Thus, the forces for the design stage might be read directly from the finite element model.

The results of the design are then compared with results derived from Traditional Design Method (TM), which relies on designing the t-section as a whole. The calculations are made for both positive and negative bending moment (often called sagging and hogging, respectively). The samples were taken from the tests performed on the scaled down girders. Additionally, the ratio of the designed to the tested reinforcement amount was calculated.

In the case of sagging, the results for RSM and TM were very similar (the same reinforcement amount required). In the hogging case, on the other hand, the RSM appeared to be a little more conservative than the TM, giving the ratio of the designed to the tested reinforcement amount equal 3.3 for RSM against 2.6 for TM.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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