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Consideration of foundation stiffness in the design of slender reinforced concrete columns in single-story frame structures

Uwzględnienie sztywności posadowienia w projektowaniu smukłych słupów żelbetowych w halach parterowych

DOI: 10.15199/33.2024.02.

Abstract. In the design of single-story frame structures, it is important to consider the foundation stiffness, which affects the value of bending moments in columns. The article presents a method and examples of second-order calculations that take into account the effect of support stiffness on the distribution of bending moments in columns supported on footings and foundation piles. The presented method allows more precise, safe and optimal design of slender reinforced concrete columns.
Keywords: slender reinforced concrete column; second-order effects; foundation stiffness; pile foundation.

Streszczenie. W projektowaniu konstrukcji parterowych hal istotne jest uwzględnienie sztywności posadowienia, która wpływa na wartość momentów zginających w słupach. W artykule przedstawiono metodę i przykłady obliczeń drugiego rzędu uwzględniające wpływ sztywności podpory na rozkład momentów zginających w słupach wspartych na stopach oraz palach fundamentowych. Przedstawiona metoda pozwala na bardziej precyzyjne, bezpieczne i optymalne projektowanie smukłych słupów żelbetowych.
Słowa kluczowe: smukły słup żelbetowy; efekty drugiego rzędu; sztywność posadowienia; fundament palowy.

Over the past two decades, the demand for halls with large volumes used as either warehousesor logistics has increased significantly. The load-bearing structure of the roof-covering in these facilities is often made up of reinforced concrete columns (photo). A major factor for the increase in the number of halls under construction is the availability of new investment areas connected with the development of road networks. Often these are areas with difficult geotechnical conditions (e.g. wasteland, agricultural), which raises the issue of designing slender reinforced concrete column systems on both highly susceptible soils and on weak-bearing soils with the need for additional reinforcements in the form of piles.

The provisions of PN-B-03264:2002 limited the slenderness of columns to a value of $l_0/i = 104$ (l_0 – effective length; i – radius of inertia of the section) and did not require consideration of the effect of ground susceptibility on the increments of bending moments. With the introduction of the Eurocodes, there has been a removal of slenderness restrictions, which, combined with the use of increasingly better materials, makes it possible to design slimmer columns than before. This requires more thorough analyses and taking into account soil-structure interaction according to section 5.8.7 of Eurocode 2 [1].

Slender reinforced concrete column
Photo: A. Sikorski
Smukłe słupy żelbetowe *Fot. A. Sikorski*

The effect of slenderness in the calculation of reinforced concrete columns has been the subject of many scientific publications, e.g. [2-6]. As a result of column deflection due to first- and second-order effects, there are increments in the eccentricities of longitudinal forces and thus increments in bending moments in the columns. Rotation of the foundation causes additional increments of moments, which can be significant in some structures. The article presents a method for considering the stiffness of foundations founded directly and on piles, which can be used in the design of reinforced concrete columns.

Calculations of the increments of moments associated with second-order effects can be performed using a simplified or general method. In the examples presented in the article, the calculations were performed using the general method involving second-order analysis with nominal stiffnesses (calculation methods are described in papers [7 – 9]).

How to consider ground susceptibility in the design of columns

In order to take into account the rotation of the footing in the static calculation of the framework, it is proposed to take as the column support model a fictitious rod with a scheme as in Fig. 1c with length L and stiffness EI. The angle of rotation at the support of the fictitious rod caused by the moment M can be calculated from the formula:

$$\varphi = ML/3EI \qquad (1)$$

while the angle of rotation of the foundation on the Winkler ground (Fig. 1a) from the formula:

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$$\varphi = M/I_F C_z \qquad (2)$$

in which:

I_F – moment of inertia of the foundation base;
C_z – coefficient of ground elasticity.

Using formulas (1) and (2), we obtain a relationship that allows us to determine the flexural stiffness of the fictitious rod:

$$M/\varphi = 3EI/L = I_F C_z \qquad (3)$$

The coefficient C_z depends not only on the physical properties of the soil but also on the dimensions of the foundation. Using the work of Gorbunov-Posadov [10] and Levinsky [11], we considered the soil as a homogeneous elastic half-space with characteristics defined by the modulus E_0 and Poisson's ratio ν_0, coefficient C_z can be calculated from the relationship:

$$C_z = \frac{\pi E_0 b' l^2}{4 I_F (1-\nu_0^2)} \qquad (4)$$

where:

l – half the length of the foundation;
b' – width of the foundation.

Taking into account

$$I_F = \frac{b'(2l)^3}{12} = \frac{2}{3} b' l^3$$

the formula is obtained:

$$C_z = \frac{3\pi}{8l} \frac{E_0}{(1-\nu_0^2)} \qquad (5)$$

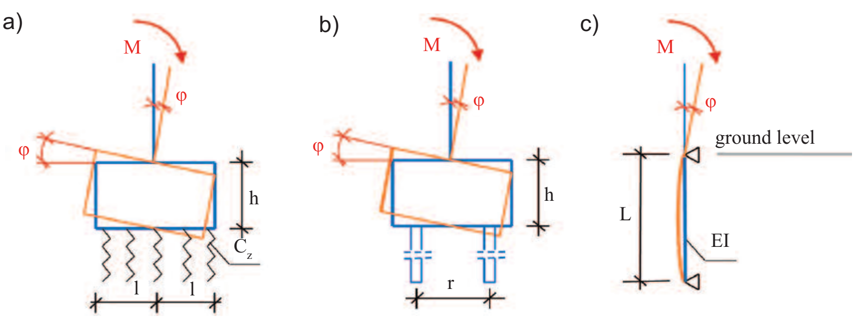

Fig. 1. Foundation on elastic subgrade (a); pile foundation (b); the way of modelling the influence of subgrade in static calculations of the foundation (c)
Rys. 1. Fundament na podłożu sprężystym (a); fundament na palach (b); sposób modelowania wpływu podłoża w obliczeniach statycznych fundamentu (c)

The above approach applies to the ground treated as a homogeneous elastic half-space. In the case of an arbitrarily layered subsoil, the value of the support rotation angle can be obtained from static calculations of the footing by any proven method. The procedure is analogous to that described below for a pile foundation. In the case of a pile foundation, the foundation rotation angle φ does not directly depend on the stiffness of the foundation. The value of the rotation angle φ is obtained as a result of calculating the displacement of individual piles in the foundation, with their distribution and interaction taken into account, Fig. 2.

The computational model of the pile support requires the adoption of a geotechnical profile along the entire length of the pile and below the footing of the pile, divided into layers with appropriately selected parameters. The main problem will be the selection of an appropriate method for calculating the settlement of piles individually and in a group. The obtained rotation angle φ allows the static calculations of the column taking into account the second-order theory.

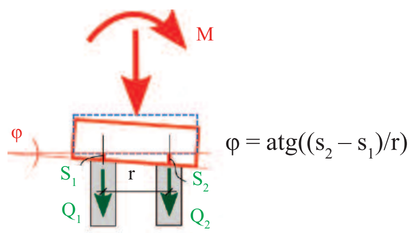

Fig. 2. Scheme for calculating settlement and rotation of pile foundation; Q_1, s_1 – load and settlement of pile 1; Q_2, s_2 – load and settlement of pile 2; r – axial spacing of piles
Rys. 2. Schemat do obliczeń osiadania i obrotu fundamentu palowego: Q_1, s_1 – obciążenie i osiadanie pala 1; Q_2, s_2 – obciążenie i osiadanie pala 2; r – rozstaw osiowy pali

Including the stiffness of the pile support in the calculation algorithm leads to new values of internal forces and nodal forces (reactions). The described procedure requires the use of iterative calculations according to the following scheme:

A) static calculations of the structural system of the hall with the assumption of restraint of columns in its supports, nodal forces are obtained;

B) determination of loads on the pile Q_i, verification of the limit state of bearing capacity of the pile, calculation of pile settlement and foundation rotation (Fig. 2);

C) static calculations of the structural system of the hall according to the second-order theory, taking into account the rotational stiffness of pile supports, new values of nodal forces are obtained;

D) determination of new loads on the pile Q_i, verification of the ultimate limit state of the pile, calculation of pile settlement and foundation rotation, as in step (B).

Steps (B) – (D) are performed until the results converge; typically, calculations require two or three iterations.

Method of calculating settlement of pile foundation

Pile foundation settlement refers to the settlement of each individual pile and the interaction with neighbouring piles. A foundation pile loaded with an axial force transfers the load to the surrounding soil medium via resistance mobilized at the side (friction) and at the base of the pile (pressure resistance), as shown in Fig. 3.

The magnitude of the resistance mobilized at the pile shaft in each layer and under the base can be related to the magnitude of the displacement (strain). This relationship is described by transformation functions τ-z and σ-z (Fig. 3). Many proposals have been developed for load transfer functions described mathematically on the basis of empirically obtained results and theoretical solu-

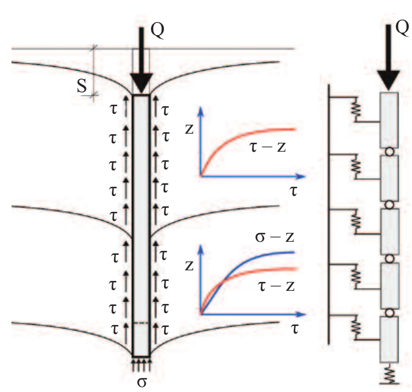

Fig. 3. Design model in the analysis of piles loaded with axial force
Rys. 3. Model obliczeniowy w analizie pali obciążonych siłą osiową

tions [12-19]. In the algorithm of the numerical method, the pile described by Young's modulus of elasticity is modelled with the useof elements with elastic supports at the nodes. The characteristics of the supports are determined by a load transfer function that reflects the mobilized resistance of the soil subsoil, Figure 3. As a result of the numerical analysis, the values of the forces generated at each node and their displacements are obtained. Calculations for successive load increments allow us to obtain nonlinear settlement (load) curves, which characterize the behaviour of the pile in the full range of its loading, Fig. 4. In addition, the separation of the total axial force into base resistance and

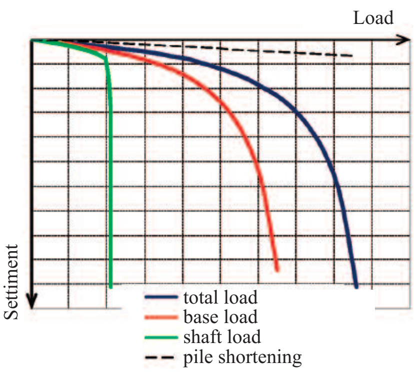

Fig. 4. Example of settlement curves for a single pile
Rys. 4. Przykład krzywych osiadania w przypadku pala pojedynczego

shaft resistance is obtained. The result is obtained from numerical analyses [15, 20, 31]. The calculation of a pile foundation according to the above scheme abandons the classical concept of pile bearing capacity in favour of predicting the pile settlement curve, i.e. establishing its load-settlement (Q-s) characteristics. Such a design concept is more rational than the classical approach and provides useful data, for example, in the design of warehouse hall columns.

In foundations that are a group of piles, the interaction between piles should additionally be taken into account [20 – 24]. The settlement of adjacent piles results from the settlement of a single pile loaded with axial force and the interaction between piles. If the foundation rests on a group of *n* piles then the settlement of the *i-th* pile taking into account the interaction between the piles can be calculated according to the formula:

$$s_{G,i} = s_i + \sum_{j=1}^{n} (s_{i,j} - s_i), \text{ for } j \neq i \quad (6)$$

where:
s_i – settlement of a single pile *i* under load Q_i;
s_{ij} – settlement of pile pair *i* and *j* loaded with forces Q_i and Q_j.

This results in higher settlements of piles in a group (s_G) compared to the settlements of the same piles considered individually (s_i, s_j).

To calculate pile settlement in a group, a method was developed [20] based on the hybrid method presented by Chow [24]. For single piles, the method used a solution to create nonlinear settlement curves for foundation piles. The method was developed and improved on the basis of analyses of the results of studies of nonlinear soil behaviour and test loads on piles under field conditions. Based on this work, an in-house computer program written in FORTRAN was developed and applied to the calculation of the examples presented in the article.

The single pile analysis method uses a nonlinear-elastic-plastic model that reflects the nonlinearity of the pile's behaviour before reaching the ultimate bearing capacity in the pile-soil contact zone. The nonlinear behaviour of the pile was described by functions describing the change (degradation) in the value of the shear modulus as a function of mobilized soil resistance or deformation. For the shaft of the pile, the characterization of the supports is based on the solution proposed by Randolph and Wroth [14].

The deflection of the support under the base of the pile (base settlement) s_b under the force P_b based on the Bousinesq solution of the theory of elasticity is described by the equation:

$$s_b = \frac{P_b}{R_b G_b} \frac{(1-\nu_b)}{4} \mu_d \quad (7)$$

where:
R_b – pile base radius;
G_b – shear strain modulus of the soil under the base of the pile;
ν_b – Poisson's ratio;
μ_d – impact factor of base depth, $\mu_d = 0{,}5$.

In the separated soil layer, the shear modulus G decreases with increasing load according to the assumed degradation function. The variation function of G-modulus has been the subject of many research projects and scientific publications. The issues of interpretation of tests for determining the initial shear modulus G_{max} and the course of its degradation are presented in [25, 26].

To describe the mechanism of degradation of soil strain modulus, a function by Fahey and Carter [27] was used to describe the tangent value of shear modulus decreasing as a function of the increment of mobilized soil resistance at the shaft:

$$G = G_{\max} \frac{\left(1-\left(\frac{\tau}{\tau_f}\right)^w\right)^2}{1-(1-w)\left(\frac{\tau}{\tau_f}\right)^w} \quad (8)$$

gdzie:
G_{max} – initial shear modulus;
τ – current mobilized soil resistance;
τ_f – limit soil resistance (at failure);
w – parameter of the equation (w = 0,2 ÷ 1,0).

To describe the behaviour of the base, functions by Van Impe and De Clercq [28] were used:

$$G = \begin{cases} G_{\max} \text{ when } \leq 10^{-5} \\ -G_{\max}(0.3 \cdot \log\gamma + 0.5) \text{ gdy } \gamma \in (10^{-5}, 10^{-2}) \\ 0.1 \cdot G_{\max} \text{ when } \gamma \geq 10^{-5} \end{cases} \quad (9)$$

where: γ – shear strain.

Figure 5 shows graphs of selected degradation functions. The use of a numerical procedure makes it possible to determine the load-settlement curve for a single pile. In addition, the results allow division and analysis of the total load into the part carried by the sidewall and the base, for successive degrees of load on the pile head. The computational model of the ground is characterized by geotechnical layers, which are described by initial shear modulus G_{max}, limt shear resistance at the shaft τ_f and limt

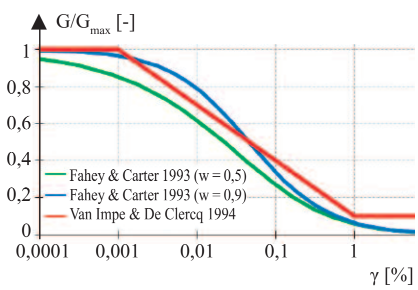

Fig. 5. Comparison of selected functions of shear modulus degradation
Rys. 5. Porównanie wybranych funkcji degradacji modułu ścinania

resistance under the pile base q_f. These parameters can be determined by an indirect method based on the results of a CPT static probe test [29 – 31]. The interaction between piles is taken into account by using the classical solution of elasticity theory relating to the analysis of points inside an elastic half--space (Mindlin's problem).

Examples

The following are example calculations of bending moments in reinforced concrete frame columns (RM-WIN program was used). The calculations were performed using the rigorous method described in the paper [9], which consists of a second-order analysis with nominal stiffnesses. Nominal stiffnesses (Section 5.8.7.2 of Eurocode 2 [1]) depend on the amount of reinforcement. In the design of columns, reinforcement must be assumed to allow the calculation of the enlarged bending moment and, based on these moments, the reinforcement needed. Thus, the result is obtained by the iteration method, after a reasonable level of correspondence is obtained between the assumed and calculated reinforcement.

The calculations were carried out for both the case of full restraint of the columns and the case of fixing the column in a footing founded on a resilient foundation (example 1) and in a foundation founded on piles (example 2). The columns were assumed to have equal cross-sections b × h = 0.40 m × 0.45m, RB500 steel, C30/37 concrete

Example 1. OThe design longitudinal forces in the columns of the two-aisle hall (Figure 6) are each $P_1 = 40$ kN in the outermost columns and $P_2 = 850$ kN in the inner column. The horizontal force was assumed to be caused by wind pressure and suction is H = 35 kN. The rigid roof structure was assumed to force equal horizontal displacements of the upper ends of all columns. Reinforcement 4 ϕ 20 (A_s =12.56 cm²) was assumed in the outermost columns and in the inner columns 6 ϕ 20 (A_s = 18.84 cm²) on each side of the section, a = 4.0 cm. Foundation footings of 2.4 × 1.8 m were adopted under the outermost columns, and 3.0 × 2.0 m under the inner column, founded on cohesive, hard-plastic soil with I_L = 0.20, E_0 = 17 MPa, v = 0.32.

The moments of inertia of the footing's base field, ground compliance coefficients and flexural stiffnesses of the fictitious support model bars for the footing under the outermost column were calculated:

$I_F = (2/3)b'l^3 = (2/3)1,8 \cdot 1,2^3 = 2,07$ m⁴;

$C_z = (3\pi/8l)(E_0/(1-\nu_0^2) = 3\pi/8 \cdot 1,2$ $17/(1-0,32)^2 = 18,59$ MN/m⁴;

$3El/L = I_F C_z = 2,07 \cdot 18,59 = 38,48$ MNm.

and internal:

$I_F = (2/3)b'l^3 = (2/3)2 \cdot 1,5^3 = 4,5$ m⁴

$C_z = (3\pi/8l)(E_0/(1-\nu_0^2)) = 3\pi/8 \cdot 1,5$ $17/(1-0,32)^2 = 14,87$ MN/m⁴

$3El/L = I_F C_z = 4,5 \cdot 14,87 = 66,92$ MNm.

Imperfections according to section 5.2 of Eurocode 2 [1]

$\alpha_h = 2/\sqrt{l} = 2/\sqrt{7,5} = 0,730$

$\alpha_m = \sqrt{0,5(1+1/m)} = \sqrt{0,5(1+1/3)} = 0,816$.

The angle of inclination of the columns

$\theta_i = \theta_0 \alpha_h \alpha_m = (1/200)0,730 \cdot 0,816 = 0,00298$.

Horizontal forces caused by imperfections:

in the outermost columns

$H_1 = \theta_i P_1 = 0,00298 \cdot 400 = 1,19$ kN;

in the centre columns

$H_2 = \theta_i P_2 = 0,00298 \cdot 850 = 2,53$ kN.

Design modulus of elasticity of concrete E_{cd} = 26670 MPa, moment of inertia $I_c = 3,0375 \times 10^{-3}$ m⁴. Coefficients k_1 i k_2 according to section p. 5.8.7.2 of Eurokod 2 [1]:

$k_1 = \sqrt{f_{ck}/20} = \sqrt{30/20} = 1,225$. For $l_0 = 2l_{col} = 2 \cdot 7,5 = 15,0$; the radius of inertia $i = h/2\sqrt{3} = 0,45/2\sqrt{3} = 0,1299$ m. Slenderness $\lambda = l_0/l_1 = 15,0/0,1299 = 115,5$. Assumed effective creep coefficient $\varphi_{ef} = 2,4$.

In the outermost columns:

$n = N_{Ed}/(A_c f_{cd}) = 400/(0,40 \times 0,45 \times 21,43 \times 10^3) = 0,104$,

$k_2 = n \cdot \lambda/170 = 0,104(115,5/170) = 0,071 \le 0,20$.

Moment of inertia of reinforcement

$I_s = 2A_s(h/2-a_1)^2 = 2 \times 12,56 \times 10^{-4}$ $(0,45/2-0,040)^2 = 8,60 \times 10^{-5}$ m⁴.

Coefficient $K_c = k_1 k_2/(1+\varphi_{ef}) = 1,225 \times 0,071/(1+2,4) = 0,026$ i $K_s = 1$.

Nominal stiffness of edge columns:

$EI = K_c E_{cd} I_c + K_s E_s I_s = 0,026 \times 26670 \times 3,0375 \times 10^{-3} + 1,0 \times 200 \times 10^3 \times 8,60 \times 10^{-5} = 19,31$ MNm².

In an internal column:

$n = N_{Ed}/(A_c f_{cd}) = 850/(0,40x0,45x21,43x10^3) = 0,220$;

$k_2 = n \cdot \lambda/170 = 0,220(115,5/170) = 0,15 \le 0,20$;

$K_c = k_1 k_2/(1+\varphi_{ef}) = 1,225x0,15/(1+2,4) = 0,054$ i $K_s = 1$.

Moment of inertia of reinforcement

$I_s = 2A_s(h/2-a_1)^2 = 2 \times 18,84 \times 10^{-4}$ $(0,45/2-0,040)^2 = 12,89 \times 10^{-5}$ m⁴.

Nominal internal column stiffness:

$EI = K_c E_{cd} I_c + K_s E_s I_s = 0,054 \times 26670 \times 3,0375 \times 10^{-3} + 1,0 \times 200 \times 10^3 \times 12,89 \times 10^{-5} = 30,15$ MNm².

The result of the calculation according to the first-order theory with nominal stiffnesses of the columns is shown in Figure 6a, according to the second--order theory in Figure 6b, and with consideration of ground susceptibility in Figure 6c and Table 1.

Example 2. The design longitudinal forces in the columns of the three-aisle

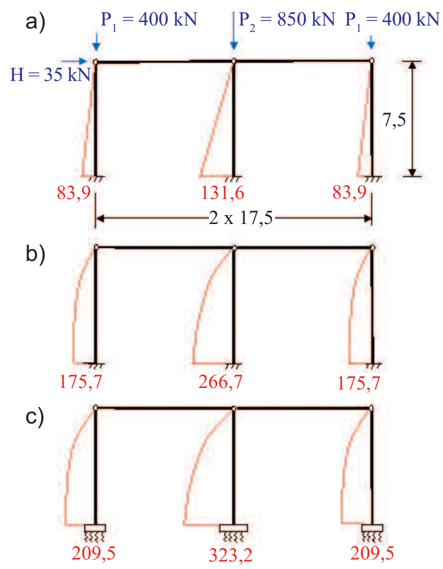

Fig. 6. Diagram of the frame of example 1 and bending moments in columns according to the first-order theory (a); moments calculated the second-order theory in case of: full restraint (b), taking subsoil susceptibility (c). The moments are given in [kNm]

Rys. 6. Schemat ramy z przykładu 1 oraz rozkład momentów zginających wg teorii I rzędu (a); momenty zginające w słupach wg teorii II rzędu w przypadku: pełnego utwierdzenia (b), posadowienia na podłożu sprężystym (c). Momenty podano w [kNm]

Table 1. Example 1 – calculation results

Tabela 1. Przykład 1 – wyniki obliczeń

Calculated quantities	Outer columns, P_1 = 400 kN support		Interior columns, P_2 = 850 kN support	
	fixed	elastic	fixed	elastic
M [kNm]	175,7	209,5	266,7	323,2
$I_F C_z = M/\varphi$ [MNm]	38,5		66,9	

hall (Figure 9) are P_1 = 500 kN in the outermost columns and P_2 = 1000 kN in the inner columns, while the the horizontal force is H = 40 kN. The eccentricity of the reaction from the overlap load in the outermost columns is equal to 0.15 m. As in example 1, the rigid roof structure forces were assumed to equal horizontal displacements of the upper ends of all columns. Reinforcement 3 ϕ 25 (A_s = 14.73 cm^2) was assumed in the outermost columns and 5 ϕ 25 (A_s = 24.55 cm^2) in the inner columns on each side of the section, a = 4.8 cm. The adopted geotechnical conditions shown in Figure 7.

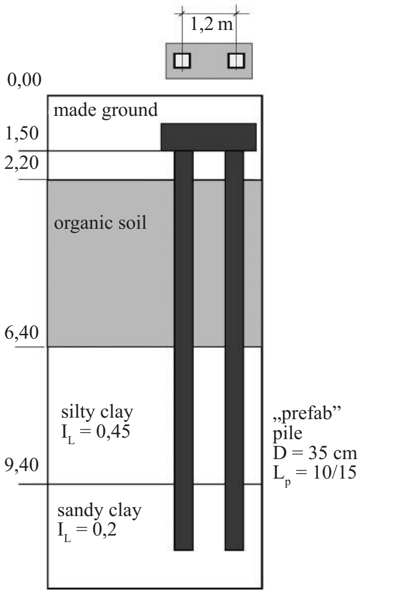

Fig. 7. Adopted design geotechnical profile
Rys. 7. Przyjęty obliczeniowy profil geotechniczny

Pile supports under the columns were adopted in the form of reinforced concrete rectangular pile caps topping the heads of two piles with axial spacing r = 1.2 m. The type of piles was reinforced concrete piles, prefabricated, driven, with a square cross-section of 0,35 × 0,35 m. The lengths of the piles (L_p = 10 and 15 m) were selected according to the assumptions of the limit state method so as to obtain a similar degree of load-bearing reserve in each case; this ensures that the limit state condition of load-bearing piles loaded with pressing forces is met. The parameters for calculating the pile foundation are summarized in Table 2. The conditions were the same for all columns. Figure 8 shows the pile settlement curves obtained from the calculations by the presented method.

Table 2. Parameters adopted for pile settlement calculations
Tabela 2. Parametry przyjęte do obliczeń osiadania pali

Name ground	Degree of plasticity I_L [-]	Unit limit resistance [kPa]	G_{max} [MPa]
Made ground	–	τ_f = 5	18
Organic	–	τ_f = 5	7
Silty clay	0,45	τ_f = 35	40
Sandy clay	0,20	τ_f = 53 q_f = 1800	54

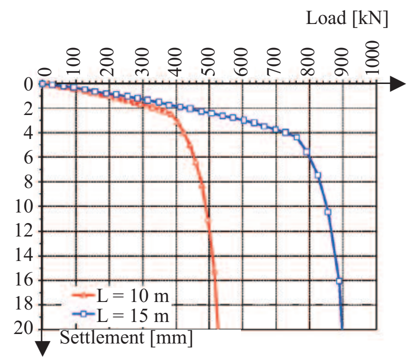

Fig. 8. Q-s settlement curves obtained for single piles
Rys. 8. Otrzymane krzywe osiadania Q-s pali pojedynczych

The calculation of the stiffness of the column-foundation connection was based on the calculation of pile settlements in the column support, taking into account the interaction between them according to the method outlined above.

Imperfections according to section 5.2 of Eurocode 2 [1]:

$\alpha_h = 2/\sqrt{l} = 2/\sqrt{8,0} = 0,707$,

$\alpha_m = \sqrt{0,5(1 + 1/m)} = \sqrt{0,5(1+ 1/4)} = 0,790$.

The angle of inclination of the columns

$\theta_i = \theta_0 \alpha_h \alpha_m = (1/200)\ 0,707 \cdot 0,790 = 0,00279$.

Horizontal forces caused by imperfections:

in the outermost columns

$H_1 = \theta_i P_1 = 0,00279 \cdot 500 = 1,395$ kN;

in the center columns

$H_2 = \theta_i P_2 = 0,00279 \cdot 1000 = 2,790$ kN;

The result of the calculation according to first-order theory with nominal column stiffnesses is shown in Figure 9b. The values of nodal reactions obtained assuming full restraint in the support are used to calculate the forces and displacements of the pile support. The calculations are carried out in an iterative manner. Table 3 summarizes the calculation results for the first and last iterations, and the final values of moments in the supports are shown in Figure 9c.

The examples show calculations for different ways of foundation of reinforced

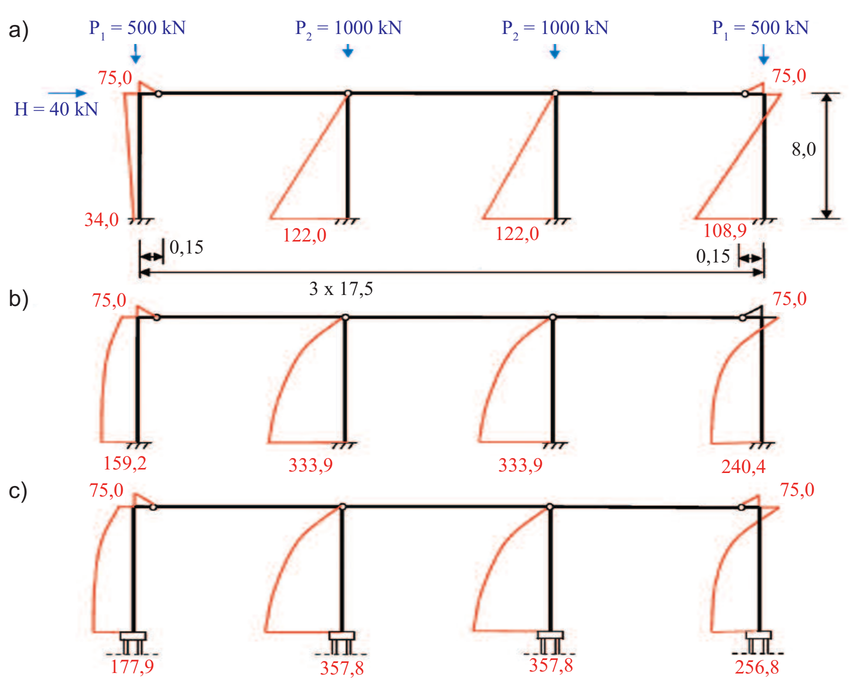

Fig. 9. Diagram of the frame of example 2 and bending moments in columns according to the first-order theory (a); moments calculated the second-order theory in case of: full restraint (b); pile foundation (c). The moments are given in [kNm]
Rys. 9. Schemat ramy z przykładu 2 oraz rozkład momentów zginających wg teorii I rzędu (a); momenty zginające w słupach wg teorii II rzędu w przypadku: utwierdzenia pełnego (b); posadowienia na palach (c). Momenty podano w [kNm]

Table 3. Example 2 – results of pile settlement calculations in column supports
Tabela 3. Przykład 2 – wyniki obliczenia osiadania pali w podporach słupów

Calculated quantities	Outer-left columns L_p = 10,0 m, V = P_1 = 500 kN		Interior columns L_p = 15,0 m, V = P_2 = 1000 kN		Outer-right poles L_p = 10,0 m, V= P_1 = 500 kN	
	iteraced 1	last iteraced	iteraced 1	last iteraced	iteraced 1	last iteraced
M [kNm]	159,2	177,9	333,9	357,8	240,4	256,8
Q_1 [kN]	226	219	419	409	193	186
Q_2 [kN]	359	367	697	707	393	400
s_1 [mm]	1,45	1,41	2,37	2,32	1,30	1,27
s_2 [mm]	2,29	2,35	3,95	4,03	2,59	2,68
Angle φ [rad]	0.00077	0.00078	0.00131	0.00142	0.00108	0.00117
M/φ [MNm]	227	227	254	251	224	219

concrete columns. In the first example – direct foundation, the differences in bending moments between the case of full restraint and the case of strict calculation including rotation of the footing exceed 19%. In the second example – foundation on piles, these differences are smaller. As a result of taking into account the foundation on susceptible pile supports, the bending moment increased by about 10% in the outermost column compared to the calculations with full attachment.

Summary

The article presents a method for modelling the support of a column fixed in a footing and a cap on foundation piles. Exemplary calculations of the frames were carried out using the strict method of II-order analysis with consideration of nominal stiffnesses by modelling the support conditions of the columns as described in the article. The effect of taking foundation stiffness into account is to increase the final values of bending moments in the columns, which will be important in structural systems with slender columns founded on highly susceptible ground. The results obtained are also influenced by the load, type and geometry of the foundation and geotechnical conditions.

Piles should be designed for ultimate forces obtained according to second-order theory, especially when designing slender columns. They should be designed for both the ultimate limit state condition and the displacement limitation conditions with the possibility of obtaining the predicted Q-s settlement curves of pile supports. The presented method of calculation allows for a more rational design of similar structures compared to standard simplified methods.

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Accepted for publications: 18.01.2024 r.